

Five-Year Integrated M. Sc. Examination 2021-2022

Semester: V

Paper: MT-3-5-5

Subject: Mathematical Theory of Probability & Statistics

Time: 3 Hours

Full Marks: 40

Questions are of value as indicated in the margin

Answer any four questions

1. (a) What is the probability of getting 10 points in 3 throws of a die?
(b) There are two drawers in each of the three boxes that are identical in appearance. The first one contains a gold coin in each drawer, the second one contains a silver coin in each drawer, but the third contains a gold coin in one drawer and a silver coin in the other. A box is chosen at random, one of its drawer is opened and a gold coin is found. What is the probability that the other drawer too will have a gold coin?
4+6
2. (a) Let $A \subset B$. Show that $P(A) \leq P(B)$.
(b) The flow of traffic at a certain street crossing is such that the probability of a car's passing during any given second is p and that there is no interaction between the passing of cars at different seconds. Treating seconds as indivisible time units, and supposing that a pedestrian can cross the street only if no car is to pass during the next three seconds, find the probability that the pedestrian has to wait exactly $k=0,1,2$ and 3 seconds.
3+7
3. (a) Find the mean and variance of a Normal distribution with parameters μ and σ^2 .
(b) Suppose you have a bag of beans of which 20% are black and remainder are white. Determine the probability that a single sample of eight beans drawn from the bag will contain at least six white beans.
6+4
4. Define unbiased estimator? Is it unique? Show that the sample mean is the best linear unbiased estimator (BLUE) for the population mean.
1+2+7

5. Describe a testing procedure to test the null hypothesis $H_0: \mu = \mu_0$ based on a randomly drawn samples of size n from $N(\mu, \sigma^2)$. Consider both the cases assuming σ^2 to be known and to be unknown. Also find the $100(1 - \alpha)\%$ confidence interval for μ assuming σ^2 to be unknown.

8+2